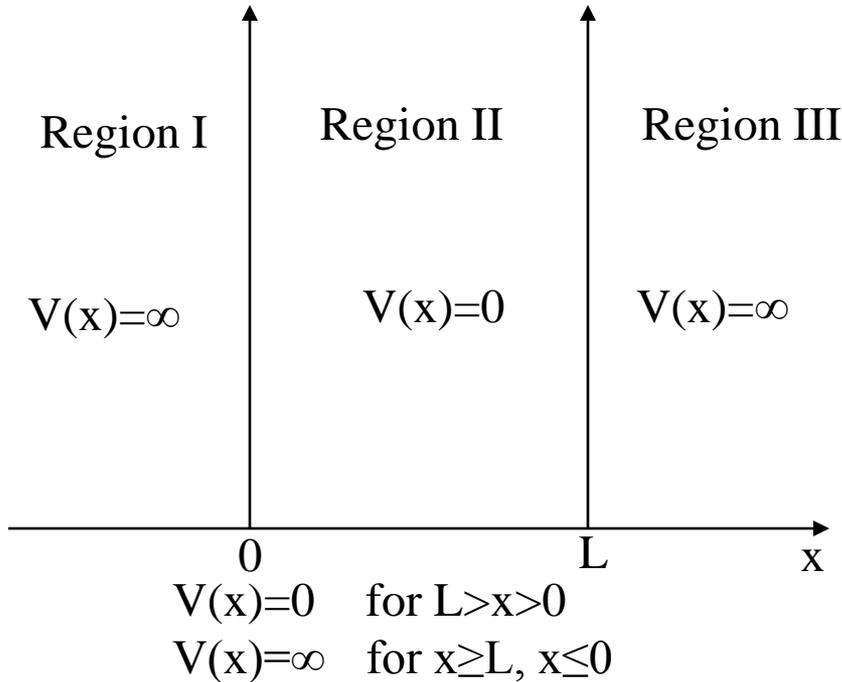


# PARTICLE IN 1-DIMENSIONAL BOX

# Particle in a 1-Dimensional Box



Classical Physics: The particle can exist anywhere in the box and follow a path in accordance to Newton's Laws.

Quantum Physics: The particle is expressed by a wave function and there are certain areas more likely to contain the particle within the box.

Time Dependent Schrödinger Equation

$$\underbrace{\frac{-\hbar^2}{2m} \frac{d^2\Psi}{dx^2}}_{\text{KE}} + \underbrace{V(x)\Psi}_{\text{PE}} = \underbrace{E\Psi}_{\text{TE}}$$

Wave function is dependent on time and position function:

$$\Psi(x, t) = f(t)\psi(x)$$

Time Independent Schrödinger Equation

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi = E\psi$$

Applying boundary conditions:

Region I and III:

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + \infty * \psi = E\psi \longrightarrow |\psi|^2 = 0$$

Region II:

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi$$

# Finding the Wave Function

$$-\frac{\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} = E\psi \longrightarrow -\frac{d^2\psi(x)}{dx^2} = \frac{2m}{\hbar^2} E\psi$$

This is similar to the general differential equation:

$$-\frac{d^2\psi(x)}{dx^2} = k^2\psi \rightarrow \psi = A \sin kx + B \cos kx$$

So we can start applying boundary conditions:

$$\begin{aligned} x=0 \psi=0 \\ 0 = A \sin 0k + B \cos 0k \rightarrow 0 = 0 + B * 1 \therefore B = 0 \end{aligned}$$

$$\begin{aligned} x=L \psi=0 \\ 0 = A \sin kL \quad A \neq 0 \rightarrow kL = n\pi \quad \text{where } n = \mathbb{N}^* \end{aligned}$$

Calculating Energy Levels:

$$k^2 = \frac{2mE}{\hbar^2} \longrightarrow E = \frac{k^2 \hbar^2}{2m} \longrightarrow E = \frac{k^2 \hbar^2}{2m 4\pi^2}$$

$$\hbar = \frac{h}{2\pi}$$

$$E = \frac{n^2 \pi^2}{L^2} \frac{\hbar^2}{2m 4\pi^2} \longrightarrow E = \frac{n^2 \hbar^2}{8mL^2}$$

Our new wave function:

$$\psi_{II} = A \sin \frac{n\pi x}{L} \quad \text{But what is 'A'?$$

Normalizing wave function:

$$\int_0^L (A \sin kx)^2 dx = 1$$

$$|A|^2 \left[ \frac{x}{2} - \frac{\sin 2kx}{4k} \right]_0^L = 1$$

$$|A|^2 \left[ \frac{L}{2} - \frac{\sin 2 \frac{n\pi}{L} L}{4 \frac{n\pi}{L}} \right] = 1$$

Since  $n = \mathbb{N}^*$

$$|A|^2 \left( \frac{L}{2} \right) = 1 \rightarrow |A| = \sqrt{\frac{2}{L}}$$

Our normalized wave function is:

$$\psi_{II} = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

# Particle in a 1-Dimensional Box

$$\psi_{II} = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \xrightarrow{\text{Applying the Born Interpretation}} |\psi_{II}|^2 = \frac{2}{L} \left( \sin \frac{n\pi x}{L} \right)^2$$

